COMMON POOL OF GENERIC ELECTIVES (GE) Semester-VI COURSES OFFERED BY DEPARTMENT OF MATHEMATICS

Category-IV

GENERIC ELECTIVES (GE-6(i)): INTRODUCTION TO MATHEMATICAL MODELING

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/ Practice		
Introduction to Mathematical Modeling	4	3	0	1	Class XII pass with Mathematics	GE-3(i): Differential Equations

Learning Objectives: The main objective of this course is to introduce:

- Compartmental models and real-life case studies through differential equations, their applications and mathematical modeling.
- Choosing the most appropriate model from competing types that have been fitted.
- Fitting a selected model type or types to the data and making predictions from the collected data.

Learning Outcomes: The course will enable the students to:

- Learn basics of differential equations and compartmental models.
- Formulate differential equations for various mathematical models.
- Construct normal equation of best fit and predict the future values.

SYLLABUS OF GE-6(i)

UNIT-I: Compartmental Models (15 hours)

Compartmental diagram and balance law; Exponential decay, radioactive dating, and lake pollution models; Case study: Lake Burley Griffin; Drug assimilation into the blood; Case study: Dull, dizzy or dead; Exponential growth, Density-dependent growth, Equilibrium solutions and stability of logistic equation, Limited growth with harvesting.

UNIT-II: Interacting Population Models and Phase-plane Analysis (15 hours)

SIR model for influenza, Predator-prey model, Ecosystem model of competing species, and model of a battle.

UNIT-III: Analytic methods of model fitting and Simulation (15 hours)

Fitting models to data graphically; Chebyshev approximation criterion, Least-square criterion: Straight line, parabolic, power curve; Transformed least-square fit, Choosing a best model. Monte Carlo simulation modeling**:** Simulating deterministic behavior (area under a curve, volume under a surface); Generating random numbers: middle-square method, linear congruence; Simulating probabilistic behavior.

Essential Readings

1. Barnes, Belinda & Fulford, Glenn R. (2015). Mathematical Modelling with Case Studies, Using Maple and MATLAB (3rd ed.). CRC Press, Taylor & Francis Group.
2. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). A First Course in Mathematical Modeling (5th ed.). CENGAGE Learning India.

Suggestive Readings

- Albright, Brian, & Fox, William P. (2020). Mathematical Modeling with Excel (2nd ed.). CRC Press, Taylor & Francis Group.
- Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). Differential Equations and Boundary Value Problems: Computing and Modeling (5th ed.). Pearson.

Practical (30 hours)- Practical / Lab work to be performed in Computer Lab: Modeling of the following problems using Mathematica/MATLAB/Maple/Maxima/Scilab etc.

1. Plotting the solution and describe the physical interpretation of the Mathematical Models mentioned below:
   a. Exponential decay and growth model.
   b. Lake pollution model (with constant/seasonal flow and pollution concentration).
   c. Case of single cold pill and a course of cold pills.
   d. Limited growth of population (with and without harvesting).
   e. Predatory-prey model (basic volterra model, with density dependence, effect of DDT, two prey one predator).
   f. Epidemic model of influenza (basic epidemic model, contagious for life, disease with carriers).
   g. Ecosystem model of competing species
   h. Battle model
2. Random number generation and then use it to simulate area under a curve and volume under a surface.
3. Write a computer program that finds the least-squares estimates of the coefficients in the following models.
   a. $y = a\,x^2 + b\,x + c$
   b. $y = a\,x^n$
4. Write a computer program that uses Equations (3.4) in [3] and the appropriate transformed data to estimate the parameters of the following models.
   a. $y = b\,x^n$
   b. $y = b\,e^{a\,x}$
   c. $y = a \ln x + b$
   d. $y = a\,x^2$
   e. $y = a\,x^3$.

GENERIC ELECTIVES (GE-6(ii)): DISCRETE DYNAMICAL SYSTEMS

CREDIT DISTRIBUTION, ELIGIBILITY AND PRE-REQUISITES OF THE COURSE

Course title & Code	Credits	Credit distribution of the course			Eligibility criteria	Pre-requisite of the course
		Lecture	Tutorial	Practical/ Practice		
Discrete Dynamical Systems	4	3	0	1	Class XII pass with Mathematics	NIL

Learning Objectives: The primary objective of this course is to introduce:

- The fundamental concepts of discrete dynamical systems and emphasis on its study through several applications.
- The concepts of the fixed points, chaos and Lyapunov exponents for linear and nonlinear equations have been explained through examples.
- Various applications of chaos in higher dimensional models.

Learning Outcomes: This course will enable the students to:

- Understand the basic concepts of difference equation, chaos and Lyapunov exponents.
- Obtain fixed points and discuss the stability of the dynamical system.
- Find Lyapunov exponents, Bifurcation, and Period-doubling for nonlinear equations.
- Analyze the behavior of different realistic systems with chaos cascade.

SYLLABUS OF GE-6(ii)

UNIT-I: Discrete-time Models (12 hours)

Discrete dynamical systems concepts and examples; Some linear models: Bouncing ball, investment growth, population growth, financial, economic and linear price models; Nonlinear models: Density-dependent population, contagious-disease, economic and nonlinear price models; Some linear systems models: Prey-predator, competing species, overlapping-generations, and economic systems.

UNIT-II: Linear Equations, Systems, their Solutions and Dynamics (18 hours)

Autonomous, non-autonomous linear equations and their solutions, time series graphs; Homogenous, non-homogeneous equations and their solutions with applications; Dynamics of autonomous linear equations, fixed points, stability, and oscillation; Homogeneous, non-homogeneous linear systems and their dynamics, solution space graphs, fixed points, sinks, sources and saddles.

UNIT-III: Nonlinear Equations, their Dynamics and Chaos (15 hours)

Autonomous nonlinear equations and their dynamics: Exact solutions, fixed points, stability; Cobweb graphs and dynamics: Linearization; Periodic points and cycles: 2-cycles, m-cycles, and their stability; Parameterized families; Bifurcation of fixed points and period-doubling; Characterizations and indicators of chaos.

Practical (30 hours)- Use of Excel/SageMath/MATHEMATICA/MATLAB/Scilab Software:

1. If Rs. 200 is deposited every 2 weeks into an account paying 6.5% annual interest compounded bi-weekly with an initial zero balance:
   (a) How long will it take before Rs. 10,000/- is in account?
   (b) During this time how much is deposited and how much comes from interest?
   (c) Create a time series graph for the bi-weekly account balances for the first 40 weeks of saving scenario.
   [1] Computer Projects 2.5 pp. 68
2. (a) How much can be borrowed at an annual interest rate of 6% paid quarterly for 5 years in order to have the payments equal Rs. 1000/- every 3 months.
   (b) What is the unpaid balance on this loan after 4 years.
   (c) Create a time series graph for the unpaid balances each quarter for the loan process.
   [1] Computer Projects 2.5 pp. 68
3. Four distinct types of dynamics for any autonomous linear equation:
   $x_{n+1} = a\, x_n + b$ for different values of a and b.
   [1] Dynamics of autonomous linear equation, pp. 74
4. Find all fixed points and determine their stability by generating at least the first 100 iterates for various choices of initial values and observing the dynamics
   a. $I_{n+1} = I_n - r\, I_n + s\, I_n\,(1 - I_n\, 10^{-6})$
      for: (i) $r = 0.5, s = 0.25$, (ii) $r = 0.5, s = 1.75$, (iii) $r = 0.5, s = 2.0$.
   b. $P_{n+1} = \frac{1}{P_n} + 0.75\, P_n + c$
      for: $(i)\ c = 0$; $(ii)\ c = -1$; $(iii)\ c = -1.25$; $(iv)\ c = -1.38$.
   c. $x_{n+1} = a\, x_n\,(1 - x_n^2)$
      for: $(i)\ a = 0.5$; $(ii)\ a = 1.5$; $(iii)\ a = 2.25$; $(iv)\ a = 2.3$.
   [1] Computer Projects 3.2 pp. 110
5. Determine numerically whether a stable cycle exists for the given parameter values, and if so, its period. Perform at least 200 iterations each time and if a cycle is found (approximately), use the product of derivatives to verify its stability.
   a. $P_{n+1} = r\, P_n\left(1 - \frac{P_n}{5000}\right)$, for: $(i)\ r = 3.4$; $(ii)\ r = 3.5$; $(iii)\ r = 3.566$; $(iv)\ r = 3.569$; $(v)\ r = 3.845$.
   b. $P_{n+1} = r\, P_n\, e^{-P_n/1000}$
      for: $(i)\ r = 5$; $(ii)\ r = 10$; $(iii)\ r = 14$; $(iv)\ r = 14.5$; $(v)\ r = 14.75$.
   [1] Computer Projects 3.5 pp. 154
6. Find through numerical experimentation the approximate intervals of stability of the (a) 2-cycle; (b) 4-cycle; (c) 8-cycle; (d) 16-cycle; (e) 32-cycle for the following
   a. $f_r(x) = r\, x\, e^{-x}$
   b. $f_r(x) = r\, x^2\,(1 - x)$
   c. $f_a(x) = x\,(a - x^2)$
   d. $f_c(x) = \frac{2}{x} + 0.75\, x - c$

[1] Computer Projects 3.6 pp. 164

7. Through numerical simulation, show that each of the following functions undergoes a period doubling cascade:
    a. $f_r(x) = r\,x\,e^{-x}$
    b. $f_r(x) = r\,x^2\,(1-x)$
    c. $f_r(x) = r\,x\,e^{-x^2}$
    d. $f_r(x) = \frac{r\,x}{(x^2+1)^2}$
    e. $f_a(x) = \;x\,(a-x^2)$

   [1] Computer Projects 3.7 pp. 175

8. Discuss (a) Pick two initial points close together, i.e., that perhaps differ by 0.001 or 0.00001, and perform at least 100 iterations of $x_{n+1} = f(x_n)$. Do solutions exhibit sensitive dependence on initial conditions?

   (b) For several random choices of x_0 compute at least 1000 iterates x_n and draw a frequency distribution using at least 50 sub-intervals. Do dense orbits appear to exit?

   (c) Estimate the Lyapunov exponent L by picking several random choices of x_0 and computing $\frac{1}{N}\sum_{n=1}^{N}\ln|f'(x_n)|$ for $N = 1000, 2500, 5000, etc.$

   Does L appear to be positive? i). $f(x) = 2 - x^2$ ii). $f(x) = \frac{2}{x} + \frac{3x}{4} - 2.$

   [1] Computer Projects 3.8 pp. 187

9. Show that $f(x) = r\,x\,(1-x)$ for $r > 4$ and $f(x) = 6.75\,x^2\,(1-x)$ have horseshoes and homoclinic orbits, and hence chaos. **[1] Computer Projects 3.8 pp. 188**

10. Find the fixed point and determine whether it is a sink, source or saddle by iterating and graphing in solution space the first few iterates for several choices of initial conditions.
    a. $x_{n+1} = x_n - y_n + 30$
       $y_{n+1} = x_n + y_n - 20.$
    b. $x_{n+1} = x_n + y_n$
       $y_{n+1} = x_n - y_n.$

   [1] Computer Projects 4.2 pp. 207

Essential Reading

1. Marotto, Frederick R. (2006). Introduction to Mathematical Modeling Using Discrete Dynamical Systems. Thomson, Brooks/Cole.

Suggestive Readings

- Devaney, Robert L. (2022). An Introduction to Chaotic Dynamical Systems (3rd ed.). CRC Press Taylor & Francis Group, LLC.
- Lynch, Stephen (2017). Dynamical Systems with Applications using Mathematica® (2nd ed.). Birkhäuser.
- Martelli, Mario (1999). Introduction to Discrete Dynamical Systems and Chaos. John Wiley & Sons, Inc., New York.